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Non-factorizable contributions in B decays *

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Abstract

First of all, $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays are studied phenomenologically and possible lower bounds of the branching ratios, $B(\bar{B}^0 \rightarrow D^0\pi^0)$ and $B(\bar{B}^0 \rightarrow D^{*0}\pi^0)$, are estimated from existing experimental data on branching ratios for the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays.

Then, $\bar{B} \rightarrow D\pi$, $D^*\pi$, $J/\psi\bar{K}$ and $J/\psi\pi$ decays are studied by decomposing their amplitude into a sum of factorizable and non-factorizable ones. The former is estimated by using the naive factorization while the latter is calculated by using a hard pion approximation in the infinite momentum frame.

The result is compared with the above phenomenological branching ratios (and their observed ones). The non-factorizable amplitude is rather small in the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays but can still efficiently interfere with the main (factorized) amplitude. In the color suppressed $\bar{B} \rightarrow J/\psi\bar{K}$ and $J/\psi\pi$ decays, non-factorizable contribution is more important. A sum of the factorized and non-factorizable amplitudes can improve the result from the factorization, although the amplitudes for the color suppressed $\bar{B}^0 \rightarrow D^0\pi^0$, $D^{*0}\pi^0$ and $\bar{B} \rightarrow J/\psi\bar{K}$, $J/\psi\pi$ decays still include ambiguities arising from uncertainties of form factors involved.

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I. INTRODUCTION

Nonleptonic weak decays of charm and B mesons have been studied extensively [1,2] by using the so-called factorization (or vacuum insertion) prescription [3]. It has been supported by two independent arguments. One is the large N_c (color degree of freedom) argument [4] that the factorizable amplitude which is given by the leading terms in the large N_c expansion dominates in hadronic weak decays. The other is that it can be a good approximation under a certain kinematical condition [5], i.e., a heavy quark decays into another heavy quark plus a pair of light quark and anti-quark which are emitted colinearly with sufficiently high energies, for example, like $b \rightarrow c + (\bar{u}d)_1$, where $(\bar{u}d)_1$ denotes a color singlet $(\bar{u}d)$ pair.

We, first, consider two body decays of charm mesons to check whether the large N_c argument works well or not in hadronic weak interactions, since the large N_c argument is independent of flavors, i.e., if it does not work in charm decays, it does not work also in B decays. A naive application of the factorization prescription to charm decay amplitudes leads to the color suppression [suppression of color mismatched decays, $D^0 \rightarrow \bar{K}^0 \pi^0$, $\bar{K}^{*0} \pi^0$, etc., described by $c \rightarrow (s\bar{d})_1 + u$]. It means, for example, that the amplitude for the decays of charm mesons into isospin $I = \frac{1}{2}$ ($\bar{K}\pi$) final states is approximately cancelled by the one into $I = \frac{3}{2}$ final states and hence the phases of these amplitudes are nearly equal to each other. Therefore the factorized amplitudes for two body decays of charm mesons should be approximately real except for the overall phase. However the observed decay rates for these decays are not always suppressed and the amplitudes for $D \rightarrow \bar{K}\pi$ and $\bar{K}^*\pi$ decays have large phase differences between the amplitudes for decays into the $I = \frac{1}{2}$ and $I = \frac{3}{2}$ final states [6]. To get rid of this problem, the factorization has been implemented by taking account for final state interactions. However, amplitudes with final state interactions are given by non-leading terms in the large N_c expansion. Therefore the large N_c argument does not work well in charm decays and hence also in B decays.

It appears that, in $\bar{B} \rightarrow D\pi$ [and $D^*\pi$] decays, the color suppression works well and the phase differences between amplitudes for decays into $I = \frac{1}{2}$ and $I = \frac{3}{2}$ final states are small. To see it explicitly, we parameterize the amplitudes for these decays as

$$A(\bar{B}^0 \rightarrow D^{[*]+}\pi^-) = \sqrt{\frac{1}{3}}A_3^{[*]}e^{i\delta_3^{[*]}} + \sqrt{\frac{2}{3}}A_1^{[*]}e^{i\delta_1^{[*]}}, \quad (1)$$

$$A(\bar{B}^0 \rightarrow D^{[*]0}\pi^0) = -\sqrt{\frac{2}{3}}A_3^{[*]}e^{i\delta_3^{[*]}} + \sqrt{\frac{1}{3}}A_1^{[*]}e^{i\delta_1^{[*]}}, \quad (2)$$

$$A(B^- \rightarrow D^{[*]0}\pi^-) = \sqrt{3}A_3^{[*]}e^{i\delta_3^{[*]}}, \quad (3)$$

where $A_{2I}^{[*]}$'s and $\delta_{2I}^{[*]}$'s are isospin eigen amplitudes for the $\bar{B} \rightarrow D^{[*]}\pi$ decays and their phases, respectively. Taking positive values of the ratio of isospin eigen amplitudes, $r^{[*]} = A_3^{[*]}/A_1^{[*]}$, we obtain

$$\cos(\delta^{[*]}) = \left(\frac{9R_{0-}^{[*]} - 1}{4}\right)r^{[*]} - \frac{1}{r^{[*]}}, \quad (4)$$

where

$$\delta^{[*]} = \delta_1^{[*]} - \delta_3^{[*]} \quad \text{and} \quad r^{[*]} = \sqrt{\frac{1}{3R_{00}^{[*]}R_{0-}^{[*]} + 3R_{0-}^{[*]} - 1}}. \quad (5)$$

Here $R_{0-}^{[*]}$ and $R_{00}^{[*]}$ are ratios of decay rates,

$$R_{0-}^{[*]} = \frac{\Gamma(\bar{B}^0 \rightarrow D^{[*]+}\pi^-)}{\Gamma(B^- \rightarrow D^{[*]0}\pi^-)} \quad \text{and} \quad R_{00}^{[*]} = \frac{\Gamma(\bar{B}^0 \rightarrow D^{[*]0}\pi^0)}{\Gamma(\bar{B}^0 \rightarrow D^{[*]+}\pi^-)}. \quad (6)$$

Values of $R_{0-}^{[*]}$ and $R_{00}^{[*]}$ can be estimated phenomenologically from the experimental data [7] on branching ratios for $\bar{B} \rightarrow D^{[*]}\pi$ decays in Table II as

$$R_{0-} = 0.58 \pm 0.10, \quad R_{00} < 0.04, \quad (7)$$

$$R_{0-}^* = 0.61 \pm 0.08, \quad R_{00}^* < 0.16. \quad (8)$$

However, these values of R_{0-} and R_{00} [R_{0-}^* and R_{00}^*] are not always compatible with each other. If all the above values of $R^{[*]}$'s are accepted, then the right-hand-side (r.h.s.) in Eq.(4) is not always less than unity. It is satisfied in more restricted regions of $R^{[*]}$, i.e., approximately, $0.04 \gtrsim R_{00} \gtrsim 0.02$, $0.68 \gtrsim R_{0-} \gtrsim 0.61$ for the $\bar{B} \rightarrow D\pi$ decays and $0.16 \gtrsim R_{00}^* \gtrsim 0.02$, $0.69 \gtrsim R_{0-}^* \gtrsim 0.53$ for the $\bar{B} \rightarrow D^*\pi$ decays. These values of $R^{[*]}$ lead approximately to the following phenomenological branching ratios, $0.34 \gtrsim B(\bar{B}^0 \rightarrow D^+\pi^-)_{\text{ph}} \gtrsim 0.28$ and $0.012 \gtrsim B(\bar{B}^0 \rightarrow D^0\pi^0)_{\text{ph}} \gtrsim 0.007$, and $0.044 \gtrsim B(\bar{B}^0 \rightarrow D^{*0}\pi^0)_{\text{ph}} \gtrsim 0.004$, when the experimental data, $B(B^- \rightarrow D^0\pi^-)_{\text{expt}} = 0.53 \pm 0.05$ and $B(B^- \rightarrow D^{*0}\pi^-)_{\text{expt}} = 0.46 \pm 0.04$, are fixed. Here we put $B(\bar{B}^0 \rightarrow D^{*+}\pi^-)_{\text{ph}} = B(\bar{B}^0 \rightarrow D^{*+}\pi^-)_{\text{expt}}$ since $\cos(\delta^*) \leq 1$ is satisfied for all the experimentally allowed values of R_{0-}^* . The allowed values of $\cos(\delta)$ are limited within a narrow region, $0.96 \lesssim \cos(\delta) \leq 1$ in which $\cos(\delta)$ is very close to unity while $\cos(\delta^*)$ is a little more mildly restricted (at the present stage) compared with the above $\cos(\delta)$, i.e., approximately, $0.70 \lesssim \cos(\delta^*) \leq 1$. Therefore, the color suppression works well, at least, in $\bar{B} \rightarrow D\pi$ decays and the phase difference δ is very small. In this way, it will be understood that the factorized amplitudes are dominant only in some specific decays like the $\bar{B} \rightarrow D\pi$ decays but it is still a little ambiguous in the $\bar{B} \rightarrow D^*\pi$ decays.

In this article, we study $\bar{B} \rightarrow D\pi$, $D^*\pi$, $J/\psi\bar{K}$ and $J/\psi\pi$ decays. In the next section, we will present our basic assumption and review briefly the effective weak Hamiltonian. In Sec. III, the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays will be studied by decomposing their amplitude into a sum of factorizable and non-factorizable ones. The former will be estimated by using the naive factorization while the latter is calculated by using a hard pion approximation in the infinite momentum frame. In Sec. IV, the color suppressed decays, $\bar{B} \rightarrow J/\psi\bar{K}$ and $B^- \rightarrow J/\psi\pi^-$, will be investigated in the same way. A brief summary will be given in the final section.

II. BASIC ASSUMPTION AND EFFECTIVE WEAK HAMILTONIAN

Our starting point to study nonleptonic weak processes is to decompose their amplitude into a sum of *factorizable* and *non-factorizable* ones [8]. Therefore, the effective weak Hamiltonian should be divided into the corresponding parts,

$$H_w = (H_w)_{\text{FA}} + (H_w)_{\text{NF}}, \quad (9)$$

where $(H_w)_{\text{FA}}$ and $(H_w)_{\text{NF}}$ are responsible for factorizable and non-factorizable amplitudes, respectively. The factorizable amplitude is estimated by using the naive factorization [1,3].

Then, assuming that the non-factorizable amplitude is dominated by dynamical contributions of various hadrons [9] and using a hard pion approximation in the infinite momentum frame (IMF) [10,11], we estimate the non-factorizable amplitudes. The hard pion amplitude will be given by *asymptotic* matrix elements of $(H_w)_{\text{NF}}$ [matrix elements of $(H_w)_{\text{NF}}$ taken between single hadron states with infinite momentum].

Before we study amplitudes for B decays, we review briefly the $|\Delta B| = 1$ effective weak Hamiltonian. Its main part is usually written in the form

$$H_w = \frac{G_F}{\sqrt{2}} V_{ud} V_{bc} \{c_1 O_1 + c_2 O_2\} + h.c., \quad (10)$$

where the four quark operators O_1 and O_2 are given by products of color singlet left-handed currents,

$$O_1 =: (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}: \quad \text{and} \quad O_2 =: (\bar{c}u)_{V-A}(\bar{d}b)_{V-A}:. \quad (11)$$

V_{ij} denotes a CKM matrix element [12] which is taken to be real since CP invariance is always assumed in this paper.

When we calculate the factorizable amplitudes for the $\bar{B} \rightarrow D^{[*]}\pi$ decays later, we use, as usual, the so-called BSW Hamiltonian [1,2]

$$H_w^{BSW} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cb} \{a_1 O_1 + a_2 O_2\} + h.c. \quad (12)$$

which can be obtained from Eq.(10) by using the Fierz reordering. The operators O_1 and O_2 in Eq.(12) should be no longer Fierz reordered. We, therefore, replace $(H_w)_{\text{FA}}$ by H_w^{BSW} . The coefficients a_1 and a_2 are given by

$$a_1 = c_1 + \frac{c_2}{N_c}, \quad a_2 = c_2 + \frac{c_1}{N_c}, \quad (13)$$

where N_c is the color degree of freedom.

When H_w^{BSW} is obtained, an extra term which is given by a color singlet sum of products of colored currents,

$$\tilde{H}_w = \frac{G_F}{\sqrt{2}} V_{ud} V_{cb} \{c_2 \tilde{O}_1 + c_1 \tilde{O}_2\} + h.c., \quad (14)$$

comes out, where

$$\tilde{O}_1 = 2 \sum_a : (\bar{c}t^a b)_{V-A}(\bar{d}t^a u)_{V-A}: \quad \text{and} \quad \tilde{O}_2 = 2 \sum_a : (\bar{c}t^a u)_{V-A}(\bar{d}t^a b)_{V-A}: \quad (15)$$

with the generators t^a of the color $SU_c(N_c)$ symmetry. To describe physical amplitudes for B decays by matrix elements of \tilde{H}_w , soft gluon(s) should be exchanged between quarks which belong to different meson states. Therefore, amplitudes given by \tilde{H}_w correspond to non-leading terms in the large N_c expansion and not factorizable so that $(H_w)_{\text{NF}}$ is now replaced by \tilde{H}_w .

III. $\bar{B} \rightarrow D\pi$ AND $D^*\pi$ DECAYS

The factorization prescription in the BSW scheme leads to the following factorized amplitude, for example, for the $B^-(p) \rightarrow D^0(p')\pi^-(q)$ decay,

$$\begin{aligned} M_{\text{FA}}(B^-(p) \rightarrow D^0(p')\pi^-(q)) \\ = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud} \left\{ a_1 \langle \pi^-(q) | (\bar{d}u)_{V-A} | 0 \rangle \langle D^0(p') | (\bar{c}b)_{V-A} | B^-(p) \rangle \right. \\ \left. + a_2 \langle D^0(p') | (\bar{c}u)_{V-A} | 0 \rangle \langle \pi^-(q) | (\bar{d}b)_{V-A} | B^-(p) \rangle \right\}. \end{aligned} \quad (16)$$

Factorizable amplitudes for the other $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays also can be calculated in the same way. To evaluate these amplitudes, we use the following parameterization of matrix elements of currents in Ref. [2],

$$\langle \pi(q) | A_\mu | 0 \rangle = -i f_\pi q_\mu, \quad (17)$$

$$\langle D(p') | V_\mu | \bar{B}(p) \rangle = \left\{ (p+p')_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right\} F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2), \quad (18)$$

$$\begin{aligned} \langle D^*(p') | A_\mu | \bar{B}(p) \rangle = & \left\{ (m_B + m_{D^*}) \epsilon_\mu^*(p') A_1(q^2) - \frac{\epsilon^*(p') \cdot q}{m_B + m_{D^*}} (p+p')_\mu A_2(q^2) \right. \\ & \left. - 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q_\mu A_3(q^2) \right\} + 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q_\mu A_0(q^2), \end{aligned} \quad (19)$$

where $q = p - p'$ and the form factors satisfy

$$A_3(q^2) = \frac{m_B + m_{D^*}}{2m_{D^*}} A_1(q^2) - \frac{m_B - m_{D^*}}{2m_{D^*}} A_2(q^2), \quad (20)$$

$$F_1(0) = F_0(0), \quad A_3(0) = A_0(0). \quad (21)$$

To get rid of useless imaginary unit except for the overall phase in the amplitude, however, we adopt the following parameterization of matrix element of vector current [13],

$$\langle V(p') | V_\mu | 0 \rangle = -i f_V m_V \epsilon_\mu^*(p'). \quad (22)$$

As stressed in Ref. [13], the above matrix element of vector current can be treated in parallel to those of axial vector currents in Eq.(17) in the infinite momentum frame (IMF). Using these expressions of current matrix elements, we obtain the factorized amplitudes for $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays in Table I, where we have put $m_\pi^2 = 0$.

Before we evaluate numerically the factorized amplitudes, we study non-factorizable amplitudes for $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays. To this, we assume that the non-factorizable amplitudes are dominated by dynamical contributions of various hadron states. Then they can be estimated by using a hard pion technique in the IMF; i.e., $\mathbf{p} \rightarrow \infty$ [10,11]. It is an innovation of the old soft pion technique [14]. In our hard pion approximation, the non-factorizable amplitude for the $\bar{B}(p) \rightarrow D^{[*]}(p')\pi(q)$ decay is given by

$$M_{\text{NF}}(\bar{B} \rightarrow D^{[*]}\pi) \simeq M_{\text{ETC}}(\bar{B} \rightarrow D^{[*]}\pi) + M_{\text{S}}(\bar{B} \rightarrow D^{[*]}\pi). \quad (23)$$

The equal-time commutator term (M_{ETC}) and the surface term (M_{S}) are given by

Table I. Factorized amplitudes for $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays where $m_\pi^2 = 0$. The CKM matrix elements are factored out.

Decay	A_{FA}
$\bar{B}^0 \rightarrow D^+\pi^-$	$i\frac{G_F}{\sqrt{2}}a_1f_\pi(m_B^2 - m_D^2)F_0^{DB}(0)\left[1 - \left(\frac{a_2}{a_1}\right)\left(\frac{f_B}{f_\pi}\right)\left(\frac{m_D^2}{m_B^2 - m_D^2}\right)\frac{F_0^{D\pi}(m_B^2)}{F_0^{DB}(0)}\right]$
$\bar{B}^0 \rightarrow D^0\pi^0$	$-i\frac{G_F}{2}a_2f_Dm_B^2F_0^{\pi B}(m_D^2)\left[1 + \left(\frac{f_B}{f_D}\right)\left(\frac{m_D^2}{m_B^2}\right)\frac{F_0^{D\pi}(m_B^2)}{F_0^{\pi B}(m_D^2)}\right]$
$B^- \rightarrow D^0\pi^-$	$i\frac{G_F}{\sqrt{2}}a_1f_\pi(m_B^2 - m_D^2)F_0^{DB}(0)\left[1 + \left(\frac{a_2}{a_1}\right)\left(\frac{f_D}{f_\pi}\right)\left(\frac{m_B^2}{m_B^2 - m_D^2}\right)\frac{F_0^{\pi B}(m_D^2)}{F_0^{DB}(0)}\right]$
$\bar{B}^0 \rightarrow D^{*+}\pi^-$	$-i\frac{G_F}{\sqrt{2}}a_1f_\pi A_0^{D^*B}(0)\left[1 - \left(\frac{a_2}{a_1}\right)\left(\frac{f_B}{f_\pi}\right)\frac{A_0^{D^*\pi}(m_B^2)}{A_0^{D^*B}(0)}\right]2m_{D^*}\epsilon^*(p') \cdot p$
$\bar{B}^0 \rightarrow D^{*0}\pi^0$	$i\frac{G_F}{2}a_2f_{D^*}F_1^{\pi B}(m_D^{*2})\left[1 + \left(\frac{f_B}{f_{D^*}}\right)\frac{A_0^{D^*\pi}(m_B^2)}{F_1^{\pi B}(m_D^{*2})}\right]2m_{D^*}\epsilon^*(p') \cdot p$
$B^- \rightarrow D^{*0}\pi^-$	$-i\frac{G_F}{\sqrt{2}}a_1f_\pi A_0^{D^*B}(0)\left[1 + \left(\frac{a_2}{a_1}\right)\left(\frac{f_{D^*}}{f_\pi}\right)\frac{F_1^{\pi B}(m_D^{*2})}{A_0^{D^*B}(0)}\right]2m_{D^*}\epsilon^*(p') \cdot p$

$$M_{ETC}(\bar{B} \rightarrow D^{[*]}\pi) = \frac{i}{f_\pi} \langle D^{[*]} | [V_\pi, \tilde{H}_w] | \bar{B} \rangle \quad (24)$$

and

$$M_S(\bar{B} \rightarrow D^{[*]}\pi) = -\frac{i}{f_\pi} \left\{ \sum_n \left(\frac{m_{D^{[*]}}^2 - m_B^2}{m_n^2 - m_B^2} \right) \langle D^{[*]} | A_{\bar{\pi}} | n \rangle \langle n | \tilde{H}_w | \bar{B} \rangle \right. \\ \left. + \sum_\ell \left(\frac{m_{D^{[*]}}^2 - m_B^2}{m_\ell^2 - m_{D^{[*]}}^2} \right) \langle D^{[*]} | \tilde{H}_w | \ell \rangle \langle \ell | A_{\bar{\pi}} | \bar{B} \rangle \right\}, \quad (25)$$

respectively, where $[V_\pi + A_\pi, \tilde{H}_w] = 0$ has been used. (See Refs. [10] and [11] for notations.) The equal-time commutator term, M_{ETC} , has the same form as the one in the old soft pion approximation but now has to be evaluated in the IMF. The surface term, M_S , is given by a divergent of matrix element of T -product of axial vector current and \tilde{H}_w taken between $\langle D^{[*]} |$ and $| \bar{B} \rangle$. However, in contrast with the soft pion approximation, contributions of single meson intermediate states can now survive when complete sets of energy eigen states are inserted between these two operators, and give a correction to the soft pion approximation. (See, for more details, Refs. [10] and [11].) Therefore, M_S is given by a sum of all possible pole amplitudes, i.e., n and l in Eq.(25) run over all possible single meson states, not only ordinary $\{q\bar{q}\}$, but also hybrid $\{q\bar{q}g\}$, four-quark $\{qq\bar{q}\bar{q}\}$, glue-balls, etc. However, n and l as well as the external states are energy eigen states in the present case so that the states which sandwich \tilde{H}_w should conserve their spin in the rest frame. Since we consider Lorentz invariant amplitudes, we should pick up n and l which conserve their spin [15], although the amplitudes are now treated in the IMF. Therefore, we drop, for example, vector meson pole contributions to the u -channel of pseudo scalar meson decays into two pseudo scalar meson states, although they have been taken into account for long time [16].

Since the B meson mass m_B is much higher than those of charm mesons and since wave function overlappings between the ground-state $\{q\bar{q}\}_0$ and excited-state-meson states are expected to be small, however, excited meson contributions will be small in these decays and can be safely neglected. Therefore the hard pion amplitudes as the non-factorizable long

distance ones are approximately described in terms of *asymptotic ground-state-meson matrix elements* (matrix elements taken between single ground-state-meson states with infinite momentum) of V_π , A_π and \tilde{H}_w .

Amplitudes for dynamical processes of hadrons can be decomposed into (*continuum contribution*) + (*Born term*). Since M_S is given by a sum of pole amplitudes, M_{ETC} corresponds to the continuum contribution [17] which can develop a phase relative to the Born term. Therefore we parameterize the ETC terms using isospin eigen amplitudes and their phases. Since the $D\pi$ final state can have isospin $I = \frac{1}{2}$ and $\frac{3}{2}$, we decompose M_{ETC} 's as

$$M_{\text{ETC}}(\bar{B}^0 \rightarrow D^+\pi^-) = \sqrt{\frac{1}{3}}M_{\text{ETC}}^{(3)}e^{i\tilde{\delta}_3} + \sqrt{\frac{2}{3}}M_{\text{ETC}}^{(1)}e^{i\tilde{\delta}_1}, \quad (26)$$

$$M_{\text{ETC}}(\bar{B}^0 \rightarrow D^0\pi^0) = -\sqrt{\frac{2}{3}}M_{\text{ETC}}^{(3)}e^{i\tilde{\delta}_3} + \sqrt{\frac{1}{3}}M_{\text{ETC}}^{(1)}e^{i\tilde{\delta}_1}, \quad (27)$$

$$M_{\text{ETC}}(B^- \rightarrow D^0\pi^-) = \sqrt{3}M_{\text{ETC}}^{(3)}e^{i\tilde{\delta}_3}, \quad (28)$$

where $M_{\text{ETC}}^{(2I)}$'s are the isospin eigen amplitudes with isospin I and $\tilde{\delta}_{2I}$'s are the corresponding phase shifts introduced. In the present approach, therefore, the final state interactions are included in the non-factorizable amplitudes. This is compatible with the fact that amplitudes with final state interactions are given by quark line diagrams which belong to non-leading terms in the large N_c expansion.

Asymptotic matrix elements of V_π and A_π are parameterized as

$$\langle \pi^0 | V_{\pi^+} | \pi^- \rangle = \sqrt{2} \langle K^+ | V_{\pi^+} | K^0 \rangle = -\sqrt{2} \langle D^+ | V_{\pi^+} | D^0 \rangle = \sqrt{2} \langle B^+ | V_{\pi^+} | B^0 \rangle = \dots = \sqrt{2}, \quad (29)$$

$$\langle \rho^0 | A_{\pi^+} | \pi^- \rangle = \sqrt{2} \langle K^{*+} | A_{\pi^+} | K^0 \rangle = -\sqrt{2} \langle D^{*+} | A_{\pi^+} | D^0 \rangle = \sqrt{2} \langle B^{*+} | A_{\pi^+} | B^0 \rangle = \dots = h, \quad (30)$$

where V_π 's and A_π 's are isospin charges and their axial counterpart, respectively. The above parameterization can be obtained by using asymptotic $SU_f(5)$ symmetry [18], or $SU_f(5)$ extension of the nonet symmetry in $SU_f(3)$. Asymptotic matrix elements of V_π between vector meson states can be obtained by exchanging pseudo scalar mesons for vector mesons with corresponding flavors in Eq.(29), for example, as $\pi^{0,-} \rightarrow \rho^{0,-}$, etc. The $SU_f(4)$ part of the above parameterization reproduces well [11,19] the observed values of decay rates, $\Gamma(D^* \rightarrow D\pi)$ and $\Gamma(D^* \rightarrow D\gamma)$.

In this way we can describe the non-factorizable amplitudes for the $\bar{B} \rightarrow D\pi$ decays as

$$M_{\text{NF}}(\bar{B}^0 \rightarrow D^+\pi^-) \simeq -i \frac{\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle}{f_\pi} \left\{ \left[\frac{4}{3}e^{i\tilde{\delta}_1} - \frac{1}{3}e^{i\tilde{\delta}_3} \right] + \dots \right\}, \quad (31)$$

$$M_{\text{NF}}(\bar{B}^0 \rightarrow D^0\pi^0) \simeq -i \frac{\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle}{f_\pi} \left\{ \frac{\sqrt{2}}{3} \left[2e^{i\tilde{\delta}_1} + e^{i\tilde{\delta}_3} \right] + \dots \right\}, \quad (32)$$

$$M_{\text{NF}}(B^- \rightarrow D^0\pi^-) \simeq i \frac{\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle}{f_\pi} \left\{ e^{i\tilde{\delta}_3} + \dots \right\}, \quad (33)$$

where the ellipses denote the neglected pole contributions.

In the case of the $\bar{B} \rightarrow D^*\pi$ decays, the matrix element $\langle V | \tilde{H}_w | P \rangle$ should vanish because of conservation of spin so that $M_{\text{ETC}}(\bar{B} \rightarrow D^*\pi)$ also should vanish but now D and B^* poles in the s - and u -channels, respectively, survive, i.e.,

$$M_{\text{NF}}(\bar{B}^0 \rightarrow D^{*+}\pi^-) \simeq \frac{i}{f_\pi} \langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle \left(\frac{m_B^2 - m_{D^*}^2}{m_B^2 - m_D^2} \right) \sqrt{\frac{1}{2}} h + \dots, \quad (34)$$

$$M_{\text{NF}}(\bar{B}^0 \rightarrow D^{*0}\pi^0) \simeq \frac{i}{\sqrt{2}f_\pi} \left[\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle \left(\frac{m_B^2 - m_{D^*}^2}{m_B^2 - m_D^2} \right) + \langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle \left(\frac{m_B^2 - m_{D^*}^2}{m_{B^*}^2 - m_{D^*}^2} \right) \right] \sqrt{\frac{1}{2}} h + \dots, \quad (35)$$

$$M_{\text{NF}}(B^- \rightarrow D^{*0}\pi^-) \simeq -\frac{i}{f_\pi} \langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle \left(\frac{m_B^2 - m_{D^*}^2}{m_{B^*}^2 - m_{D^*}^2} \right) \sqrt{\frac{1}{2}} h + \dots, \quad (36)$$

where the ellipses denote the neglected excited meson contributions. Therefore the non-factorizable amplitudes in the hard pion approximation are controlled by the asymptotic ground-state-meson matrix elements of \tilde{H}_w (and the possible phases).

Now we evaluate the amplitudes given above. The factorized amplitudes in Table I contain many parameters which have not been measured by experiments, *i.e.*, form factors, $F_0^{DB}(q^2)$, $A_0^{D^*B}(q^2)$, $F_1^{\pi B}(q^2)$, etc., and decay constants, f_D , f_D^* , f_B , etc. The form factors $F_0^{DB}(0)$ and $A_0^{D^*B}(0)$ can be calculated by using the heavy quark effective theory (HQET) [20]. The other form factors are concerned with light mesons and therefore have to be estimated by some other models. In color favored decays, main parts of the factorized amplitudes depend on the form factor, $F_0^{DB}(0)$ or $A_0^{D^*B}(0)$, and the other form factors are included in minor terms proportional to a_2 . Therefore our result may not lead to serious uncertainties although some model dependent values of the form factors are taken. (We will take, later, the values given in Ref. [21].) In the color suppressed $\bar{B}^0 \rightarrow D\pi^0$ and $D^{*0}\pi^0$ decays, however, the factorized amplitudes are proportional to the form factors, $F_0^{\pi B}(m_D^2)$ and $F_1^{\pi B}(m_{D^*}^2)$, respectively. Since their values are model dependent, the result on the color suppressed decays may be a little ambiguous, if non-factorizable contribution is less important. For the decay constants of heavy mesons, we assume $f_D \simeq f_{D^*}$ (and $f_B \simeq f_{B^*}$) since D and D^* (B and B^*) are expected to be degenerate because of heavy quark symmetry [20] and are approximately degenerate in reality. Here we take $f_{D^*} \simeq f_D \simeq 211$ MeV and $f_{B^*} \simeq f_B \simeq 179$ MeV from a recent result of lattice QCD [22]. In this way, we can obtain the factorized amplitudes in the second column of Table II, where we have neglected very small annihilation terms in the $\bar{B}^0 \rightarrow D^0\pi^0$ and $D^{*0}\pi^0$ decay amplitudes.

To evaluate the non-factorizable amplitudes, we need to know the size of the asymptotic matrix elements of \tilde{H}_w and A_π taken between heavy meson states. The latter which was parameterized in Eq.(30) is estimated to be $|h| \simeq 1.0$ [10,11] by using partially conserved axial-vector current (PCAC) and the observed rate [7], $\Gamma(\rho \rightarrow \pi\pi)_{\text{expt}} \simeq 150$ MeV. For the asymptotic matrix elements, $\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle$ and $\langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle$, we treat them as unknown parameters and search phenomenologically for their values to reproduce the observed rates for the $\bar{B} \rightarrow D^{[*]}\pi$ decays. To this, we parameterize these matrix elements using factorizable ones of H_w^{BSW} as $\langle D^{[*]0} | \tilde{H}_w | \bar{B}^{[*]0} \rangle = B_H \langle D^{[*]0} | H_w^{BSW} | \bar{B}^{[*]0} \rangle_{FA}$ where B_H is a parameter introduced and, for example,

$$\langle D^0 | H_w^{BSW} | \bar{B}^0 \rangle_{FA} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud} \left(\frac{m_D^2 + m_B^2}{2} \right) f_D f_B a_2. \quad (37)$$

In this way, we obtain the hard pion amplitudes as the non-factorizable contributions listed in the third column of Table II, where the CKM matrix elements have been factored out.

Table II. Factorized and non-factorizable amplitudes for the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays. The CKM matrix elements are factored out.

Decay	$A_{\text{FA}} (\times 10^{-5} \text{ GeV})$	$A_{\text{NF}} (\times 10^{-5} \text{ GeV})$
$\bar{B}^0 \rightarrow D^+\pi^-$	$1.54 a_1 \left\{ 1 - 0.11 \left(\frac{a_2}{a_1} \right) \right\}$	$-3.70 a_2 B_H \left\{ \left[\frac{4}{3} e^{i\tilde{\delta}_1} - \frac{1}{3} e^{i\tilde{\delta}_3} \right] \right\}$
$\bar{B}^0 \rightarrow D^0\pi^0$	$-1.29 a_2 \left\{ \frac{f_D}{0.211 \text{ GeV}} \right\}$	$-3.70 a_2 B_H \left\{ \frac{\sqrt{2}}{3} \left[2e^{i\tilde{\delta}_1} + e^{i\tilde{\delta}_3} \right] \right\}$
$B^- \rightarrow D^0\pi^-$	$1.54 a_1 \left\{ 1 + 1.18 \left(\frac{a_2}{a_1} \right) \right\}$	$3.70 a_2 B_H \left\{ e^{i\tilde{\delta}_3} \right\}$
$\bar{B}^0 \rightarrow D^{*+}\pi^-$	$-1.53 a_1 \left\{ 1 - 0.28 \left(\frac{a_2}{a_1} \right) \right\}$	$3.70 a_2 B_H \left\{ -0.694 \right\}$
$\bar{B}^0 \rightarrow D^{*0}\pi^0$	$1.10 a_2 \left\{ \frac{f_{D^*}}{0.211 \text{ GeV}} \right\}$	$3.70 a_2 B_H \left\{ +0.00135 \right\}$
$B^- \rightarrow D^{*0}\pi^-$	$-1.53 a_1 \left\{ 1 + 1.02 \left(\frac{a_2}{a_1} \right) \right\}$	$3.70 a_2 B_H \left\{ -0.696 \right\}$

We now compare our result on the branching ratios, $B(\bar{B} \rightarrow D\pi)$ and $B(\bar{B} \rightarrow D^*\pi)$, with experiments, taking a sum of the factorized amplitude (the second column in the Table II) and the non-factorizable amplitude (the third column in Table II) as the total one. To this, we determine values of parameters involved. We take $V_{cb} = 0.0395$ from the updated value $|V_{cb}| = 0.0395 \pm 0.0017$ [7]. For the coefficients a_1 and a_2 in H_w^{BSW} , we do not know their true values. According to Ref. [23], NLO corrections to a_1 are small while corresponding corrections to a_2 may be not much smaller compared with the LO corrections and depend strongly on the renormalization scheme. Therefore, we expect that the value, $a_1 = 1.024$, with the LO corrections [23] is not very far from the true value and we take conservatively the above value of a_1 . For a_2 , however, we consider two cases. We take $a_2 = 0.125$ with the LO QCD corrections [23] as the case (i) and then we treat it as an adjustable parameter around the above $a_2 = 0.125$ as the case (ii). For the phases $\tilde{\delta}_1$ and $\tilde{\delta}_3$ arising from contributions of non-resonant multi-hadron intermediate states into isospin $I = \frac{1}{2}$ and $\frac{3}{2}$ final states, they are restricted in the region $|\tilde{\delta}_{2I}| < 90^\circ$ since resonant contributions have already been extracted as pole amplitudes in M_S although their contributions are neglected as discussed before. For B_H , we here treat it as a free parameter.

We now search for values of parameters, $\tilde{\delta}_1$, $\tilde{\delta}_3$ and B_H in the case (i), and a_2 , $\tilde{\delta}_1$, $\tilde{\delta}_3$ and B_H in the case (ii), to reproduce the phenomenologically estimated branching ratios (from the observed ones) for the $\bar{B} \rightarrow D^{[*]}\pi$ decays. Large $\tilde{\delta}_1$, ($90^\circ > \tilde{\delta}_1 \gtrsim 60^\circ$), and small $|\tilde{\delta}_3|$ are favored but our result is not very sensitive to the latter. For the B_H parameter, $B_H \simeq 0.40$ in (i) but smaller values, $0.2 \gtrsim B_H \gtrsim 0.1$, in (ii) are favored. We list our results on the branching ratios in (i) $a_1 = 1.024$, $a_2 = 0.125$, $\tilde{\delta}_1 = 85^\circ$, $\tilde{\delta}_3 = -5^\circ$ and $B_H = 0.40$, and (ii) $a_1 = 1.024$, $a_2 = 0.19$, $\tilde{\delta}_1 = 85^\circ$, $\tilde{\delta}_3 = -5^\circ$ and $B_H = 0.15$ in Table III, where we have used the central values, $V_{cb} = 0.0395$, $V_{ud} = 0.98$, $\tau(B^-) = 1.65 \times 10^{-12} \text{ s}$ and $\tau(\bar{B}^0) = 1.56 \times 10^{-12} \text{ s}$, of their experimental data. B_{FA} and B_{tot} are given by the factorized amplitude and a sum of the factorized and non-factorizable ones, respectively. Values of B_{ph} have been obtained phenomenologically from B_{expt} [7] in Sec. I. B_{FA} , in which the non-factorizable contributions are neglected, can reproduce fairly well the existing data. However, if we add the non-factorizable contributions, we can improve the fit to the phenomenologically estimated B_{ph} in both cases, (i) and (ii). It is seen that the non-factorizable contributions to the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays are rather small but still can interfere efficiently with the main amplitude given by the naive factorization.

Table III. Branching ratios (%) for $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays where the central values of experimental data [7], $V_{cb} = 0.0395$, $V_{ud} = 0.98$, $\tau(B^-) = 1.65 \times 10^{-12}$ s and $\tau(\bar{B}^0) = 1.56 \times 10^{-12}$ s, have been used. $a_2 = 0.125$ with the LO QCD corrections and $B_H = 0.40$ in (i) and phenomenological $a_2 = 0.19$ and $B_H = 0.15$ in (ii) have been taken, respectively, but $a_1 = 1.024$, $\tilde{\delta}_1 = 85^\circ$, $\tilde{\delta}_3 = -5^\circ$ in both cases. B_{FA} and B_{tot} are given by the factorized amplitude and a sum of the factorized and non-factorizable ones, respectively. The values of phenomenologically estimated B_{ph} have been given in the text.

Decays		B_{FA}	B_{tot}	B_{ph}	$B_{\text{expt}} (*)$
$B(\bar{B}^0 \rightarrow D^+\pi^-)$	(i)	0.28	0.30	$0.28 - 0.34$	0.30 ± 0.04
	(ii)	0.27	0.28		
$B(\bar{B}^0 \rightarrow D^0\pi^0)$	(i)	0.003	0.011	$0.006 - 0.012$	< 0.012
	(ii)	0.007	0.012		
$B(B^- \rightarrow D^0\pi^-)$	(i)	0.40	0.49	0.53 ± 0.05	0.53 ± 0.05
	(ii)	0.46	0.51		
$B(\bar{B}^0 \rightarrow D^{*+}\pi^-)$	(i)	0.26	0.30	0.276 ± 0.021	0.276 ± 0.021
	(ii)	0.25	0.27		
$B(\bar{B}^0 \rightarrow D^{*0}\pi^0)$	(i)	0.002	0.002	$0.004 - 0.044$	< 0.044
	(ii)	0.005	0.005		
$B(B^- \rightarrow D^{*0}\pi^-)$	(i)	0.37	0.43	0.46 ± 0.04	0.46 ± 0.04
	(ii)	0.42	0.45		

IV. $\bar{B} \rightarrow J/\psi \bar{K}$ AND $J/\psi \pi$ DECAYS

Now we study Cabibbo-angle favored $\bar{B} \rightarrow J/\psi \bar{K}$ and suppressed $B^- \rightarrow J/\psi \pi^-$ decays in the same way as in the previous section. Both of them are color suppressed and their kinematical condition is much different from the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays at the level of underlying quarks, *i.e.*, $b \rightarrow (c\bar{c})_1 + s$ in the former but $b \rightarrow c + (\bar{u}d)_1$ in the latter. Therefore, dominance of factorized amplitudes in the $\bar{B} \rightarrow J/\psi \bar{K}$ and $B^- \rightarrow J/\psi \pi^-$ decays has no theoretical support and hence non-factorizable long distance contribution may be important in these decays.

The factorized amplitude for the $\bar{B} \rightarrow J/\psi \bar{K}$ decays is given by

$$M_{\text{FA}}(\bar{B} \rightarrow J/\psi \bar{K}) = -iV_{cb}V_{cs} \left\{ \frac{G_F}{\sqrt{2}} a_2 f_\psi F_1^{KB}(m_\psi^2) \right\} 2m_\psi \epsilon^*(p') \cdot p. \quad (38)$$

The value of the decay constant of J/ψ is estimated to be $f_\psi \simeq 380$ MeV from the observed rate [7] for the $J/\psi \rightarrow \ell^+\ell^-$. The value of the CKM matrix element V_{cs} is given by $V_{cs} \simeq V_{ud} \simeq 0.98$. The value of the form factor $F_1^{KB}(m_\psi^2)$ has not been measured and its theoretical estimates are model dependent. We pick out tentatively the values of $F_1^{KB}(m_\psi^2)$ based on the following five models, *i.e.*, BSW [1], GKP [24], CDDFGN [25], AW [26] and ISGW [27], and list the corresponding $B_{\text{FA}}(\bar{B} \rightarrow J/\psi \bar{K})$ in Table IV, where we have used $V_{cb} = 0.0395$, $\tau_B^- = 1.65 \times 10^{-12}$ s, $\tau_{\bar{B}^0} = 1.56 \times 10^{-12}$ s as before. For a_2 , we consider again two cases, *i.e.*,

Table IV. Branching ratios (%) for the $\bar{B} \rightarrow J/\psi \bar{K}$ decays where the values of $F_1^{KB}(m_\psi^2)$ estimated in the five models, BSW, GKP, CDDFGN, AW and ISGW, in Refs. [1], [24], [25], [26] and [27], respectively, are used. Values of the other parameters involved are the same as in Table III, where $B'_H = B_H$ has been assumed. The data values are taken from Ref. [7].

Models		BSW	GKP	CDDFGN	AW	ISGW
$F_1^{KB}(m_\psi^2)$		0.565	0.837	0.726	0.542	0.548
B_{FA}	(i)	0.015	0.032	0.024	0.014	0.014
	(ii)	0.034	0.075	0.056	0.031	0.032
B_{tot}	(i)	0.040	0.066	0.055	0.038	0.038
	(ii)	0.052	0.101	0.079	0.049	0.050
Experiment		$B(B^- \rightarrow J/\psi K^-) = (0.099 \pm 0.010) \%$ $B(\bar{B}^0 \rightarrow J/\psi \bar{K}^0) = (0.089 \pm 0.012) \%$				

(i) $a_2 = 0.125$ and (ii) $a_2 = 0.19$. The results (B_{FA}) from the factorized amplitudes for the values of $F_1^{KB}(m_\psi^2)$ listed in Table IV are considerably smaller (except for the GKP [24]) than the observations [7],

$$\begin{aligned}
B(B^- \rightarrow J/\psi K^-)_{\text{expt}} &= (0.099 \pm 0.010) \% \\
B(\bar{B}^0 \rightarrow J/\psi \bar{K}^0)_{\text{expt}} &= (0.089 \pm 0.012) \%.
\end{aligned} \tag{39}$$

Non-factorizable contributions to these decays are estimated by using a hard kaon approximation which is a simple extension of the hard pion technique in the previous section. With this approximation and isospin symmetry, non-factorizable amplitude for the $\bar{B} \rightarrow J/\psi \bar{K}$ decays is given by

$$M_{\text{NF}}(\bar{B} \rightarrow J/\psi \bar{K}) = \frac{i}{f_K} \langle \psi | H_w | \bar{B}_s^{*0} \rangle \left(\frac{m_B^2 - m_\psi^2}{m_{B_s^*}^2 - m_\psi^2} \right) \sqrt{\frac{1}{2}} h + \dots, \tag{40}$$

where the ellipsis denotes neglected contributions of excited mesons [28] and $\langle \bar{B}_s^{*0} | V_{K^+} | B^- \rangle = -1$ and $\sqrt{2} \langle \bar{B}_s^{*0} | A_{K^+} | B^- \rangle = -h$ which are flavor $SU_f(3)$ extensions of Eqs.(29) and (30) have been used. Asymptotic matrix element, $\langle \psi | \tilde{H}_w | \bar{B}_s^{*0} \rangle$, is parameterized in the same way as $\langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle$ before. Then the total amplitude for the $\bar{B} \rightarrow J/\psi \bar{K}$ decays is approximately given by

$$M_{\text{tot}}(\bar{B} \rightarrow J/\psi \bar{K}) \simeq -i V_{cb} V_{cs} \{ 5.73 F_1^{KB}(m_\psi^2) + 5.16 B'_H \} a_2 \times 10^{-5} \text{ GeV} \tag{41}$$

where $f_K \simeq 160 \text{ MeV}$ and $f_{B_s^*} \simeq f_{B_s} \simeq 204 \text{ MeV}$ from the updated lattice QCD result [22] have been taken. B'_H is a parameter corresponding to B_H , i.e.,

$$\langle \psi | \tilde{H}_w | \bar{B}_s^{*0} \rangle = B'_H \langle \psi | H_w^{BSW} | \bar{B}_s^{*0} \rangle_{\text{FA}}. \tag{42}$$

When we take $a_2 = 0.19$ and $B'_H = 0.15$ as before, we can reproduce considerably well the existing experimental data on the $\bar{B} \rightarrow J/\psi \bar{K}$ decays by B_{tot} although the result depends sharply on the values of the form factor $F_1^{KB}(m_\psi^2)$.

For the Cabibbo-angle suppressed $B^- \rightarrow J/\psi\pi^-$, the same technique and values of parameters as the above lead to

$$M_{\text{tot}}(B^- \rightarrow J/\psi\pi^-) \simeq -iV_{cb}V_{cd}\{5.73F_1^{\pi B}(m_\psi^2) + 5.46B'_H\}a_2 \times 10^{-5} \text{ GeV}. \quad (43)$$

Using $F_1^{\pi B}(m_\psi^2) \simeq F_1^{KB}(m_\psi^2)$ expected from $SU_f(3)$ symmetry, we obtain

$$B_{\text{tot}}(B^- \rightarrow J/\psi\pi^-) \simeq \left| \frac{V_{cd}}{V_{cs}} \right|^2 B_{\text{tot}}(B^- \rightarrow J/\psi K^-) \quad (44)$$

which is well satisfied by experiment [7]. $B_{\text{tot}}(B^- \rightarrow J/\psi\pi^-)$ from the amplitude Eq.(43) which includes both of the factorized amplitude and the non-factorizable one can reproduce the existing experimental data [7],

$$B(B^- \rightarrow J/\psi\pi^-)_{\text{expt}} = (5.0 \pm 1.5) \times 10^{-5}, \quad (45)$$

by taking (i) $a_2 = 0.125$ and $B'_H = 0.40$, and (ii) $a_2 \simeq 0.19$ and $B'_H = 0.15$ as before, although $B'_H = B_H$ is not necessarily required.

V. SUMMARY

In summary, we have investigated the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ and found that the existing data on their branching ratios are not always compatible with each other, i.e., the r.h.s. of Eq.(4) is over unity for some values of $R_{00}^{[*]}$ and $R_{0-}^{[*]}$. Then we have obtained phenomenologically allowed values of their branching ratios, B_{ph} , which keep the r.h.s. of Eq.(4) approximately less than unity. Next, we have studied the $\bar{B} \rightarrow D\pi$, $D^*\pi$, $J/\psi\bar{K}$ and $J/\psi\pi^-$ decays describing their amplitude by a sum of factorizable and non-factorizable ones. The former amplitude has been estimated by using the naive factorization while the latter has been calculated by using a hard pion (or kaon) approximation in the infinite momentum frame. The so-called final state interactions (corresponding to the NLO terms in the large N_c expansion) have been included in the non-factorizable long distance contributions. The non-factorizable contribution to the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays is rather small and therefore the final state interactions seem to be not very important in these decays although still not necessarily negligible. By taking $a_1 \simeq 1.024$ with the LO QCD corrections and the phenomenological $a_2 \simeq 0.19$ which has been suggested previously [21,29], the observed branching ratios for these decays can be well reproduced in terms of a sum of the hard pion amplitude and the factorized one. Namely, the factorized amplitudes are dominant but not complete and long distance hadron dynamics should be carefully taken into account in hadronic weak interactions of B mesons.

In color suppressed $\bar{B}^0 \rightarrow D^0\pi^0$, $\bar{B} \rightarrow J/\psi\bar{K}$ and $J/\psi\pi^-$ decays, non-factorizable long distance contributions are more important. In particular, in the $\bar{B} \rightarrow J/\psi\bar{K}$ decay, long distance physics should be treated carefully. When $a_2 \simeq 0.125$ with the LO QCD corrections is taken instead of the phenomenological $a_2 \simeq 0.19$, it may be hard to reproduce the observed values of $B(\bar{B} \rightarrow J/\psi\bar{K})$ and $B(B^- \rightarrow J/\psi\pi^-)$ even by taking a sum of factorized and non-factorizable amplitudes as long as $B'_H = B_H \simeq 0.4$ is taken.

The non-factorizable amplitudes are proportional to asymptotic ground-state-meson matrix elements of \tilde{H}_w , i.e., B_H or B'_H . To reproduce large rates for the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays, the non-factorizable contributions are needed ($B_H \neq 0$) while too large values of B_H and B'_H will lead to too large rates for the color suppressed decays. However, their numerical results are still ambiguous since the amplitudes for the color suppressed decays depend sharply on model dependent form factors.

Therefore more precise measurements of branching ratios for the color suppressed decays, in particular, $B(\bar{B} \rightarrow D^0\pi^0)$, are useful to determine the non-factorizable long distance contributions in hadronic weak decays of B mesons.

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